

AKRATA, FOR 16 WINDS BY IANNIS XENAKIS: ANALYSES¹

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... a sketch in which I make use
of the theory of groups ²

ABSTRACT

In the following paper the author presents a reconstruction of the formal system developed by Iannis Xenakis for the composition of *Akrata*, for 16 winds (1964-1965). The reconstruction is based on a study of both the score and the composer's preparatory sketches to his composition.

Akrata chronologically follows *Herma* for piano (1961) and precedes *Nomos Alpha* for cello (1965-1966). All three works fall within the “symbolic music” category. As such, they were composed under the constraint of having a *temporal function* unfold “algorithmically” the work's “outside time” architecture into the “in time” domain.

The system underlying *Akrata*, undergoes several mutations over the course of the music's unfolding. In its last manifestation, it bears strong resemblances with the better known system later applied to the composition of *Nomos Alpha*. These observations suggest a much more empirical approach to musical composition than the composer's published texts of that period imply.

1. INTRODUCTION

Akrata was composed over a two-year period between 1964 and 1965, placing it right after *Hiketides* (1964) for women's choir and small ensemble and before *Terretektorh* (1965-1966) for orchestra. In these two compositions Xenakis explored the possibilities of using space as an independent musical parameter. Parallel to these considerations in which mathematics intervened functionally and plastically over the unfolding of his musical material, Xenakis also pursued a very different project in which discrete algebraic entities were to form an all encompassing structural basis for an entire musical composition.

This latter approach originated in a period of intense theoretical activity started around the end of the 1950's and through which the composer thought to find, among other things, “*justification for what [he] was doing*”³. His first report on what he was to call “symbolic music” was published as the last chapter of his main theoretical work “*Musiques Formelles*”⁴ in 1963. In this work Xenakis followed the thoughts of a hypothetical listener confronted with

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² Iannis Xenakis about *Akrata* in [1] page 19.

³ In conversation with A. B. Varga : “Then came a phase when I tried to justify what I was doing” [11] page 82.

⁴ See “*musique symbolique*” [15] pages 183 - 208 or [18] pages 155 - 177 for the English translation.

generic “sound events”. Through the protagonist of his *gedankenexperiment*, who has forgotten everything he ever knew about music, Xenakis postulated a radical *tabula rasa*, with the hope of unveiling the immediate and thus “unmediated” elements of sound perception. His first conclusions led him to define an *outside time* structure as comprising the algebraic properties that the different parameters of sound (pitch, intensity, duration) are endowed with when considered independently of the passing of time. Noticing that time, expressed first in terms of succession and then of duration intervals, was endowed with similar properties, he called this second structure *temporal*. Finally, in probably the boldest move in his text, he defined the *in time* structure, the one closest to an actual musical score, as resulting from a “correspondence” between the *outside time* and the *temporal* structures.

The compositional approach that evolved out of these considerations can be summarized in Xenakis’ wish, or a priori postulate, to unify a musical composition under a single, all encompassing logic. More concretely, this translated into the devising of an *outside time* structure which, once unfolded *inside time* through a *temporal function* (or *correspondence*) would generate the work (or a family of works) without the need of any outside intervention⁵. The first concrete “application” of this concept resulted in the composition of *Herma* for piano (1961). Its underlying system, however, still drew heavily on *stochastic* procedures. It is only in 1967 that Xenakis published the description of a system that much more faithfully embodied his initial project⁶. Indeed almost every aspect of *Nomos Alpha* for cello (1965-1966) can be traced back to a (mainly) deterministic model built on the unfolding “in time” of mathematical *groups*⁷.

Akrata was completed shortly before Xenakis started composing *Nomos Alpha*. As will be seen, the system that underlies the former undergoes several alterations over the course of the music’s unfolding. In its last manifestation, it bears a very strong resemblance with the principles later applied to the composition of *Nomos Alpha*. These observations invite a reconsideration of the composer’s theories from a perspective that identifies his thinking *about* music and his compositional process as two facets of a single process, without any necessary precedence of the former over the latter.

2. THE SONIC SURFACE OF *AKRATA*: A FIRST DESCRIPTION

The translation Xenakis gives of the title of his first composition for wind ensemble, *Akrata*, is “pure” (plural, neuter)⁸. A reference, maybe, to the simplicity and elegance of the principles that form the heart of the system he conceived to conjure his work into existence. Alternatively, it could also be a reference to the music’s relatively calm surface when compared with other of the composer’s works.

Akrata is scored for eight woodwinds (Piccolo, Oboe, Bb Clarinet, Eb Clarinet, Bass Clarinet, Bassoon and two Contrabassoons) and eight brass instruments (two Horns, three Trumpets, two tenor Trombones and a Tuba). If a single feature had to be selected to characterize the work’s musical surface, it would probably be its “horizontality”. Its

⁵ The composer’s first experiments in this direction, although based on stochastic procedures, can be traced back to as early as 1956 and his ST-series of algorithmic, computer generated, compositions (1956-1962). See [3] page 69-72 for a brief discussion of that point.

⁶ See “vers une philosophie de la musique” [16] pages 71 - 119 or [18] pages 201 - 241 for the English translation.

⁷ Let us recall that a *group* is a set G of elements together with a binary operation (written as \bullet) such that the four following axioms are satisfied: 1) *closure*: $a \bullet b$ belongs to G for all a and b in G ; 2) *associativity*: $(a \bullet b) \bullet c = a \bullet (b \bullet c)$ for all a, b, c in G ; 3) *identity*: there exists a unique element e in G such that $a \bullet e = e \bullet a = a$ for all a in G ; and finally 4) *inversion*: for each element a in G there exists a unique element a' in G such that $a \bullet a' = a' \bullet a = e$. The set of integers together with the binary operation of addition satisfies all the above axioms and thus form a familiar example of a *group*.

⁸ Program note to *Akrata* [13].

entire sonic material is indeed forged out of held pitches, appearing without any obvious relation to each other, and disappearing in a similar way after lasting as little as a single sixteenth note to as long as twenty five quarter notes (tempo is 60 to 75 beats per minute). The work completely evades any reference to traditional melody or harmonic progression, orienting ones ear primarily to the most immediate parameters such as timbre, dynamics and texture. The clarity with which these serve to delineation the four main sections of the work confirms their central role. Table 1 presents an overview of their evolution through the course of the work.

	Section A	Section B	Section C	Section D
Measures	1 - 56	57 - 119	120 - 199	200 – 371
Tutti				200-215 255
Duration min –max, (average) (in quarter notes)	3.5 – 8.0 (5.6)	1.5 – 18.0 (10.1)	0.25 – 4.75 (2.56)	0.75– 25.0 (10.1)
Registers	d ₂ - d ₄	b ^b ₋₁ - c ₂ ; c ₃ - b ^b ₅	b ^b _o - b ^b ₅	b ^b ₋₁ - b ₆
Orchestration	Brass only	Brass and woodwinds	Woodwinds only	Brass and woodwinds
Playing modes and textures	stacc, flatt, mute, normal No octave doublings	Normal Octave doublings	Staccato throughout No octave doubling	Stacc., flat., mute, normal Octave doublings / composite sonorities
Dynamics	<i>ppp, pp, mp, f, ff</i>	<i>ppp, pp, p, mp, mf, ff, fff</i>	<i>In order of appearance: fff (x 11), ff, f, mf, mp, p, pp, ppp, fff, ppp</i>	All dynamics crescendi and decrescendi, composite sonorities

Table 1: *Akrata* evolution of some of the musical parameters.

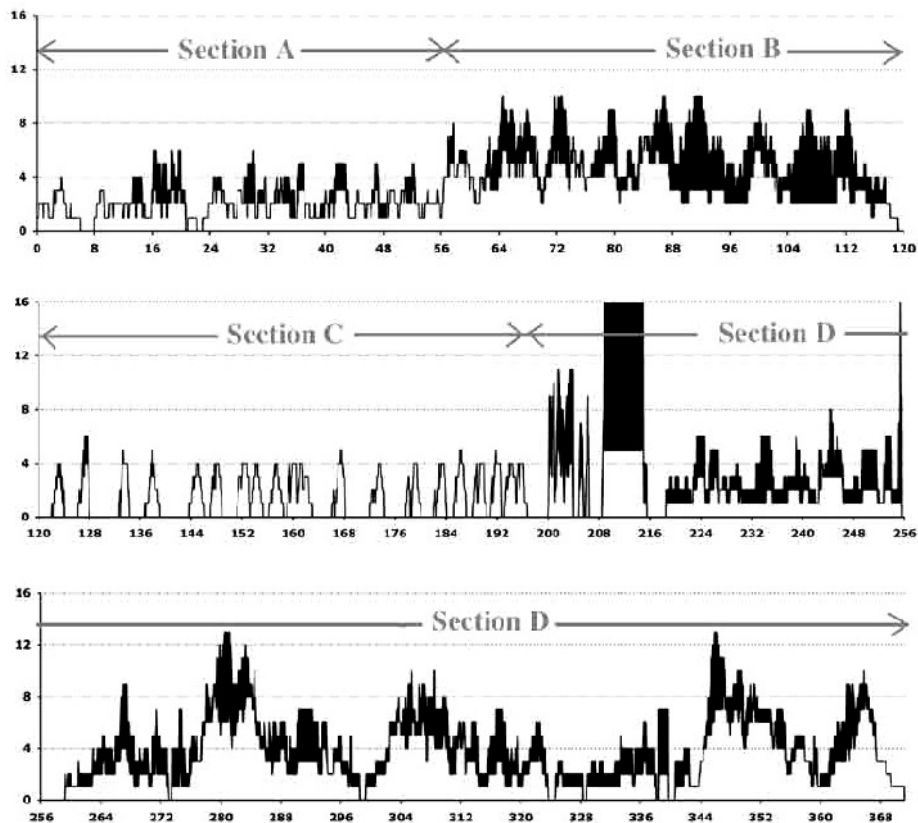
The first section of the piece presents pitches of medium duration, all played by the brass instruments within a range of just two octaves from d₂ to d₄. The first 15 measure of the work are reproduced in *Example 1* below. Playing modes alternate freely every few measures between *staccato* (dominant over the 20 opening measures), mute, flutter tongue, and normal. After the opening *mp*, dynamics markings ranging from *ppp* to *ff* alternate at a similar rate of change, and usually in concord with the changes in playing mode.

The entry of the woodwinds, at bar 57, marks the beginning of section B. While new timbral combinations are being explored, the playing mode is set at normal throughout. The pitches now cover almost the entire available range and are held on average twice as long as in the previous section. This, together with the frequent use of octave doublings results in a clear increase in overall density. The absence of silence and the continuous character of the section is balanced by a constant fluctuation of the musical surface based at least as much on timber and dynamics as on changes in the pitch material. Graph 1 below, shows the “breathing” like quality induced by the fluctuations in density. The parameter is represented in terms of the total number of pitches played at any given time (upper line) as well as the resulting number of distinct pitch classes (lower line).

An abrupt decrease in activity followed by three bars of silence mark the beginning of the next section of the work. The woodwinds, alone, now project powerful sonic outbursts of about 8 pitches each played *staccato* at *fff*. These *sound complexes* are well separated from one another by long stretches of silence As the dynamic markings gradually

decrecendo, the distance separating *sound complexes* decreases as well. This “closing in” finally accelerates until the “collision” that announces the final and longest section of *Akrata*.

This last section, the longest, is also the one where the pitches are clearly held the longest: ten quarter notes on average, going all the way to 25 quarter notes. Few, however, are simply played sustained. A wealth of orchestration techniques are used to “animate” each pitch. One can find numerous octave doubling, dove tailing from one instrument to the next, varying dynamics, creating combinations of playing modes, articulating the sound rhythmically between two or more instruments, “sculpting” their attacks, or asking the performers to create beats between close frequencies in the lower register. A short excerpt of this section is reproduced in Example 2, page 18.



Graph 1: *Akrata*, evolution of the density.

The final section of the work refers back to section B in the sense that it also projects a continuous, but ever changing musical surface with similar fluctuations in density (see Graph 1). The much longer durations that characterize the section, however, create the impression of a general slowing down of time providing the listener, metaphorically, with the leisure to contemplate each sonority in detail and to discover, within it, a hidden and marvelous activity.

Example 1: *Akrata*, measures 1 to 15, (copyright Boosey & Hawkes Music publisher, 1968).

3. PRELIMINARY REMARKS

3.1. Sources and Methodology

Xenakis has said very little about the system he applied to the composition of *Akrata*. It is probably in the commentary appearing in the program notes of the work that he is at his most expansive on the matter:

This work is of an extra temporal architecture, based on the theory of groups of transformations. Use is made on it of the theory of Sieves, a theory which annexes the modulo z congruences and which is the result of an axiomatization of music. In it, use is made of complex numbers (imaginary ones)⁹.

During his conversations with Mario Bois¹⁰, Xenakis makes a few more comments about mathematical groups in connection with *Akrata*. He does not, however, go much into details about how he conceived it. To the author's knowledge, the only other source is to be found in a footnote of *Towards a Metamusic*¹¹. In it, the composer mentions the fact that he used Fibonacci like sequences of group elements similar to the one he applied, later, to the composition of *Nomos Alpha*. As the examination of the score, of which we have just given an quick overview, reveals nothing about the work's underlying system, the only recourse was to examine Xenakis' sketches. These are located at the

⁹ Program note to *Akrata* [13].

¹⁰ See [1] page 19.

¹¹ See "vers une métamusic" [16] pages 68 or [18] pages 199 (note 25, page 376) for the English translation.

Bibliothèque Nationale de France, in Paris.

The documents relative to the composition of *Akrata* represent about a dozen files, containing numerous pages of calculations, theoretical notes, graphs, analyses etc. These are not musical sketches in the usual sense, as the pages of drawing paper greatly outnumber those of staff paper. Xenakis' theoretical considerations, calculations and graphs are scattered across the files, bearing in most cases no indication as to whether they have *anything* to do with *Akrata*. In fact, some mention other works the composer was working on at the time. Filed "as they were found", the documents do not necessarily respect any chronology. The mention of a date is about as rare as an explicit mention of the title of the work. The frequent use of different colors could not lead to any meaningful clues either, as it could be interpreted as either traces of "layers" of work appearing on a same sheet of paper, as a "color coding" of which only the composer knew the meaning or, still, merely as reflecting the fact that Xenakis liked to use different colored pens.

Reconstructing the compositional system behind *Akrata*, just like archeology, can not be considered an exact science. A few words about how the results have been validated is thus in order. The first criteria has been, of course, the consistency with the musical score. It is now well documented that Xenakis did not always follow exactly the systems he set forth¹². In most of *Akrata*, the "fit" between the system as presented here and the score could be considered as quite good. As will be discussed, difficulties arise mainly in the last section of the composition. At times, documents, containing information particularly close to what the composition "should be" have been used as a basis in lieu of and in place of the score itself.

3.2. The Rotations of the Tetrahedron

Before turning to the compositional system itself, the logical universe in which Xenakis immersed himself for the composition of *Akrata* will be shortly described. It is based on a single mathematical group. This group, which we will call T, can be thought of intuitively as the set of all rotations of a tetrahedron that appear to leave the solid fixed while permuting its vertices, or more abstractly as a certain subgroup of the group of all possible permutations of four elements. The binary operation of the group corresponds to performing rotations (or permutations) in succession. The twelve elements of the group can be subdivided into the following three categories:

The identity transformation, labeled I, leaves all the vertices as they are and corresponds to the neutral element of the group¹³.

Three transformations rotate the tetrahedron by 180° around an axis joining the centers of two opposite vertices: these are labeled A, B, and C. The transformation C is shown in Figure 1. Note that if this operation is repeated, then the edges return to their original position. This relation is expressed symbolically as $A^2 = B^2 = C^2 = I$.

Finally there are four transformations that rotate the tetrahedron by 120° around axes joining vertices to the center of their opposite face. These are labeled D, E, L, and G. These transformations can be repeated to give a 240° rotation, but repeating them once more brings the vertices back to their original position. So, where $D^2, E^2, L^2,$ and G^2 are all distinct rotations, $D^3 = E^3 = L^3 = G^3 = I$.

¹² See for instance Solomos [9] or Gibson [6].

¹³ The labels used for each rotation are the ones used by Xenakis in his sketches.

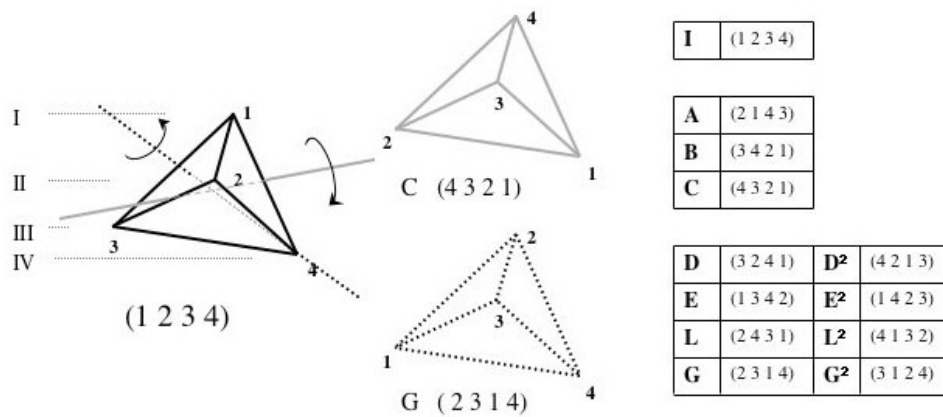


Figure 1: the group of rotations of a tetrahedron.

As shown on the right hand-side of Figure 1, the elements of T can be represented by specifying the order they induce on the vertices of the tetrahedron with respect to a fixed reference order (1 2 3 4). The induced ordering is then written in parenthesis: G becomes (2 3 1 4), and C becomes (4 3 2 1). From this perspective, each rotation is a certain permutation of four elements; that is, as the “action” of permuting four “objects” (in our case, vertices) into the order specified in parentheses. (2 3 1 4) moves the second “object” to the first place, the third to the second place, the first to the third place, and the fourth stays at the fourth place. The group operation then consists of performing two permutations in succession. It can be easily verified, and is actually intuitively quite clear, that the combination of two rotations will induce the same permutation of the vertices as combining their corresponding permutations. The table on the left hand side of Figure 2 gives the result obtained when two elements of T are combined. The first element of the operation is taken from the top and the second from the left. The result can then be read at the intersection in the usual manner. One readily notices that the group operation does not commute. For instance, combining G and B gives L^2 , while combining B and G gives E^2 .

The set of *all* the possible orderings (permutations) of four “objects” is called the *symmetric group on four elements*, denoted S_4 , and has size (or cardinality) 24, twice the size of the group T . Indeed, some permutations, (1 2 4 3) for instance, can not be obtained by rotating the tetrahedron; instead, one would also have consider reflections, such as through the planes defined by two vertices and the center of their opposite edge. The group of rotations of the tetrahedron, which we are calling T , turns out to correspond to the subgroup A_4 of so-called *even* permutations in the group S_4 .

3.3. Cycles of Group Elements in T

Taking any two elements of the group T and applying the group operation to them will determine a third element, possibly distinct from the previous two. Applying the operation to the second and the third element results in a fourth one. With the third and the fourth, one obtains a fifth, and so on. This procedure reminds us of the one for generating Fibonacci sequences: start with two numbers, and let each new number be the sum of the previous two. In the case of finite groups, however, the resulting sequence will not continue indefinitely, but will loop back onto itself as soon as the first two elements of the series reappear next to each other and in the same order. For this reason, we will refer to *cycles* of group elements rather than to *sequences*.¹⁴ For instance, starting with the couple A - D , one obtains the

¹⁴ Anyone who has read an analysis of *Nomos Alpha* (as in Solomos [9] or Vriend [12]) will be already familiar with this procedure. It is described by Xenakis in “vers une métamusique” [16] pages 38-70 or [18] pages 180-200 for the English translation.

following cycle:

$$A - D - G - E - B - G^2 - L - D - A - L^2 - E^2 - G^2 - B - E - D^2 - L^2 - (A - D)$$

One can also represent such a cycle of elements by the “path” it forms through the table of binary operations as shown on the right hand side of Figure 2. Selecting A at the top and D on the left, one starts the path at the intersection G. Taking now D on the top and G on left one obtains the next element E, and so on.

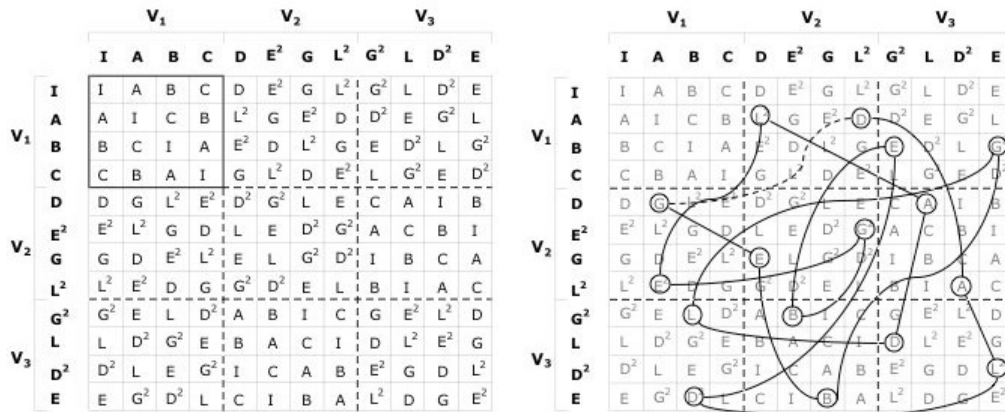


Figure 2: Table of operations in T and example of “Fibonacci cycle”.

3.4. The V_i -Cycles

Taking a closer look at the inner structure of the group T, Xenakis identifies an interesting feature¹⁵. Considering the following partition¹⁶ of T into three subsets: $V_1 = \{I, A, B, C\}$; $V_2 = \{D, G, L^2, E^2\}$; $V_3 = \{E, L, D^2, G^2\}$; he notices that no matter which elements X and Y are selected, to know the subset V_i to which their combination belongs, it is enough to know the subset to which X belongs and the subset to which Y belongs. For example, taking any two elements, one in V_1 , say A, and another in V_2 , say D, one knows that their combination (in this case, G) will be in V_2 . This assertion can be verified by examining the table of operations in Figure 2. The elements appearing in each square marked by the dotted lines do indeed belong to the same subset V_i .

These three subsets can be considered as themselves forming a group under the operation induced by the underlying elements¹⁷. The three V_i subsets forming a group, it becomes possible to generate V_i -cycles in the exact same way as the T-cycles. Taking, for instance, V_1 and V_2 as starting couple, one obtains the following:

$$V_1 - V_2 - V_2 - V_3 - V_1 - V_3 - V_3 - V_2 - (V_1 - V_2)$$

What this all implies is the following: listing, in succession, the V_i subsets to which the elements of a T-cycle belong as, for example, in Table 2, then the sequence of V_i 's obtained turns out to form a group cycle as well. A group

¹⁵ Making this assertion is not without taking some risks as no document could be found that unambiguously showed that Xenakis thought of what is to follow in precisely the terms used. Furthermore, some documents show that he explored a set of *complex functions* exhibiting a group structure isomorphic to that formed by the V_i . It was actually the author's first assumption that Xenakis used these, in line with the reference he makes to complex numbers in his program note.

¹⁶ The division of a set S into any number of subsets so that all the elements of S appear within at least, and at most, one of the subsets is called, in mathematics, a *partition* of S.

¹⁷ Mathematically, the subset V_i is a so called *normal subgroup* of T and the group $\{V_1, V_2, V_3\}$ corresponds the quotient group T / V_i .

cycle of elements of the V_i group under its own binary operation. In the case of Table 2, it is the same V_i -cycle as above. Although the T and V_i cycles are intrinsically related, they have *different* periodicities. Xenakis, as will be seen shortly, uses the T-cycles to endow progressions between basic units of his composition with a “necessity”. One can see how, musically, having two sets of distinct cardinality determine cycles with distinct periods all under the same basic logic creates both variety and economy of means.

A	D	G	E	B	G ²	L	D	A	L ²	E ²	G ²	B	E	D ²	L ²	A	D
V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂

Table 2: T and V_i cycles.

4 THE FIRST SECTION OF *AKRATA* AS AN “EXPOSITION” OF THE WORK’S LOGIC

4.1 The Logic of Succession

The cycles of elements of T are used extensively throughout the composition of *Akrata*. They form, so to speak, the abstract “backbone”, or “hidden necessity” of the work. Figure 3 shows how they determine the overall *outside time* structure of each section.

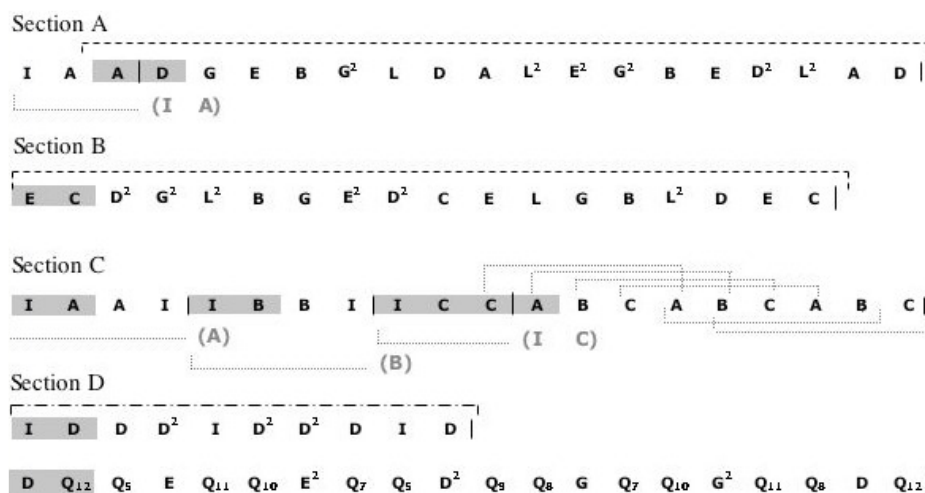


Figure 3: The underlying T-cycles of *Akrata*.

Cycles of T-elements turn out to have only four possible lengths¹⁸: 1, 3, 8, and 16. Each cycle is determined by its starting pair, for which there are 144 choices. If one considers each cycle independently of any starting point (i.e., independently of where one “cuts” open a cycle in order to set it out linearly), then there are only 16 distinct cycles: 1 of length 1, 5 of length 3, 4 of length 8 and 6 of length 16. Concerning *Akrata*, Xenakis uses all possible cycle lengths. He uses a cycle of length 3 interrupted by a cycle of length 16 in sections A, a different cycle of length 16 in section B, and several cycles of length 3 in section C almost exhausting, in the latter case, the available cycles of that length. In section D, Xenakis actually escapes from the group T. He starts with a cycle of length 8, within T, but then he follows this with a cycle of length 18 in S_4 (some elements of the cycle are in T, but not the entire cycle). This move away from the basic

¹⁸ When discussing the length of group cycles, we will not count the last two elements that are duplicates of the first two. The cycle I - A - A - I - A will thus be referred to as having length 3, not 5. The length 1 arises only from the trivial cycle, I - I - I.

logical mechanism of *Akrata*, as will be seen later, has significance within Xenakis' compositional scheme.

4.2. The translation into *sound complexes*

One of the key feature of the compositional system underlying *Akrata*, is the way in which each group element is translated into a *sound complex*. In their abstract form, these *sound complexes* consist of sets of four (from section B on eight) pitch / duration vectors (henceforth "vectors"). These are obtained in three steps. The first consists of representing geometrically each element of the group of rotations as a set of regions on a four by four grid. Remembering that a rotation is unambiguously identified once the positioning it induces on the vertices is stated; Xenakis selects the regions that connect the vertices, represented by the rows and numbered 1 to 4, to their respective positions with respect to the origin, represented by the column and numbered I to IV. Figure 4 displays this encoding with respect to rotation G.

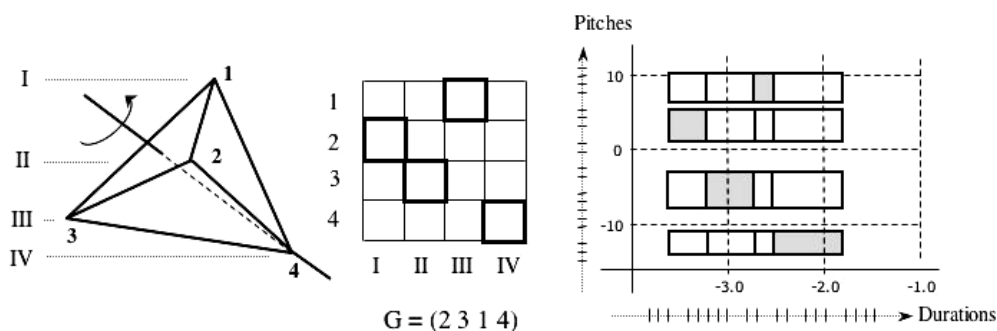


Figure 4: translation of T elements into *sound complexes* (section A).

The second stage brings us a step closer to actual musical elements. The sixteen squares of the grid are projected onto the same number of distinct areas on a Euclidian plane with duration represented on the x-axis and pitch on the y-axis as shown on the right hand-side of Figure 4. The units shown are the ones actually used by Xenakis. On the y-axis the 0 corresponds to e_3 and the basic unit is the half tone. The negative values applied to the x-axis, however surprising can easily be converted into durations by letting 0 represent 5 seconds. These sixteen regions remain fixed throughout section A. One readily recognizes the overall range delimited for this section with, in the pitch domain, -14 corresponding to d_2 and 10 to d_4 and in the duration domain -3.6 corresponding to 3.5 quarter notes and -1.8 to 8 quarter notes (with respect to a metronomic marking of 60 beat per minute).

Finally, actual values are assigned to the vectors corresponding to the element in each cycle. Their selection is ultimately left to the composer, though they are constrained in two different ways. First, Xenakis attaches a sieve¹⁹ to each group element along the cycle that delimits the range of possibilities on *both* duration and pitch. Secondly, the selection is made within the regions specified by the geometric representation of the permutation considered and projected in the obvious way on the regions of the duration / pitch plane. To each region corresponds one, and only one, vector. In the example shown in Figure 4, the vectors would thus be selected within the shaded areas and according to the sieves placed on both axes.

It should be noted that the values thus selected do, in a very concrete sense, "encode" the transformation to

¹⁹ To the reader unfamiliar with the concept of a *sieve* we suggest either Xenakis' own account in [14] pages 75-87 (268-276 in [18]) or Benoît Gibson's particularly clear article on the subject in [7]. The fact that the same *sieve* refers, in each *sound complex*, to both pitch and duration is noteworthy, as it corresponds to an idea that Xenakis was already toying with while composing *Metastasis* (1953-1954). See [2] and especially [8].

which they belong. To take a simple example, the identity element will induce a linear relationship between pitch and duration within the complex. The vector assigned the highest pitch value will also have the longest duration while the vector with the lowest pitch value will have the shortest duration. This “encoding” is *independent* of the order in which the elements of the sound complex are presented. To underline once more their relations to the rotations and to avoiding the regularities that a reading from left to right along the grid would induce, Xenakis presents the elements in an order that again reflects their underlying permutation. In the case of a sound complex attached to G, as in the example of Figure 4, the vectors appear, with respect to the numbering on the referential as II, III, I and IV.

4.3. Sieves and sieve transformations

Akrata is the very first composition in which Xenakis applied the concept of a sieve. Having conceived his work essentially within the half step equal temperament system, the possibilities it afforded him to explore alternative tuning-systems did not, at first, retain his attention²⁰. Rather, he concentrated on the possibility to mechanize their generation, creating a link with his overall logical framework. To each *sound complex*, is attached a different *sieve* derived from the previous two through intersection, union, complementation and transposition. The analogy with the underlying logic of succession, where each rotation is the result of the combination of the previous two, is not fortuitous. Indeed, Xenakis determines the operations he applies to his sieves in the first two section of his work through the V_i -cycles that run parallel to the T-cycles.

A new sieve Z is obtained by applying to the two sieves that precede it (X and Y) an operation determined by the V_i subsets attached to X, and the V_i subsets attached to Y, as shown on the left hand side of Table 3. The table gives the resulting V_i subset of Z (in accordance with the group operation) as well as its construction out of X and Y (the arrow refers to the operation of transposition).

Section A		Section B		Section C			Section D		
				Initial sieve:		$\delta_0 + A_1$	Initial sieve:		$\delta_0 \oplus = \overline{A_0 B_0 C_0}$
X - Y	Z =	X - Y	Z =	X	Y	Z	X	Y	Z
$V_1 - V_2$	\overline{Y}	$V_1 - V_2$	\overline{Y}	I	{A, B, C}	\overline{Y}	A	A	$\overline{Y} \oplus$
$V_1 - V_3$	$\overline{XY} \oplus$	$V_1 - V_3$	$\overline{XY} \oplus$	{A, B, C}	{A, B, C}	$\overline{Y} \oplus$	Q	A	$\overline{X + Y}$
$V_2 - V_1$	\overline{Y}	$V_2 - V_1$	\overline{Y}	{A, B, C}	I	\overline{Y}	A	Q	\overline{XY}
$V_2 - V_2$	$\overline{XY} \oplus$	$V_2 - V_2$	$\overline{XY} \oplus$	I	I	\overline{Y}	Q	Q	\overline{Y}
$V_2 - V_3$	$\overline{XY} \oplus$	$V_2 - V_3$	$\overline{XY} \oplus$						
$V_3 - V_1$	$\overline{XY} \oplus$	$V_3 - V_1$	\overline{Y}						
$V_3 - V_2$	$\overline{XY} \oplus$	$V_3 - V_2$	$\overline{XY} \oplus$						
$V_3 - V_3$	$\overline{XY} \oplus$	$V_3 - V_3$	$\overline{XY} \oplus$						

Table 3 : sieves and sieve transformations.

For what concerns the sieves initiating the inductive process, Xenakis considers the elementary *sieves* having periodicities 2, 3, and 5 and which he labels A, B, and C respectively. By introducing a periodicity of 5 within his construction, Xenakis actually obtains sieves with, in general, a periodicity of two and a half octaves. After having explored the entire gamut of possible combinations the composer finally selects A, B, and the following two composite sieves for his composition:

$$\delta_0 = A_0 \overline{B_0 C_0} \quad \text{and} \quad d_0' = \overline{A_0 B_0 C_0}$$

²⁰ Xenakis will, however, use such systems later on, in particular in composing of *Nomos Alpha*.

Rather than just determining the first two *sieves*, Xenakis goes all the way to the fourth: **I**: $\square_0 + A_1$; **A**: $\square_0 + A_1$; **A**: $\square_1 + A_0$; **D**: $\square_0 + B_0$, before letting the inductive process take its course.

4.4. Dynamics and playing modes

Numerous elements in the sketches point towards attempts to include the intensities and the playing modes into the composition's underlying logic. Xenakis seems, however, to have had difficulties arriving at something he felt satisfied with. Several graphs, such as the one shown in Figure 5, can be found among his notes. The exact mapping displayed below, however, could not be found among the available documents. As presented, it is a hypothetical "reconstruction" from the score as summarized in Table 4. Maybe Xenakis used one of the graphs that was actually found, such as, for instance, the one displaying the following mapping: $V_1: mp / mp$; $V_2: f / ff / f / ff$; $V_3: ppp / ff / ff / ppp$, and corrected what he obtained as he went along. Alternatively, it is also possible that somewhere, Xenakis devised a mapping that was much closer to the one of Figure 5, minimizing the a posteriori "corrections". Another possibility, of course, is that he ended up working completely intuitively on this aspect of his composition.

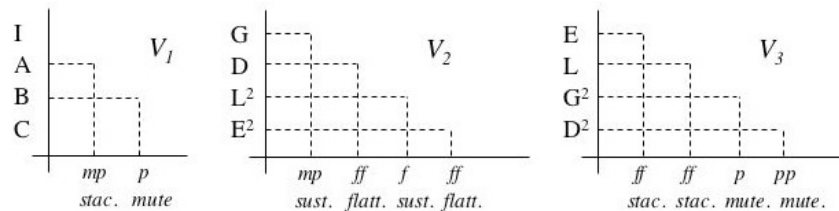


Figure 5: Determination of dynamics and playing modes.

Section A																			
I	A	A	D	G	E	B	G ²	L	D	A	L ²	E ²	G ²	B	E	D ²	L ²	A	D
		(V ₁)	(V ₂)	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂
stac	stac	stac	stac	sust.	stac	stac	mute	stac	flatt	stac	sust.	flatt	mut e	mute	stac	mute	sust.	stac	flatt
mp	mp	mp	mp	mp	ff	ff	ppp	ff	ff	mp	f	ff	p	p	ff	pp	f	Mp	Ff

Section B																	
E	C	D ²	G ²	L ²	B	G	E ²	D ²	C	E	L	G	B	L ²	D	E	
V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	
pp	ppp	mp	mp	p	pp	ff	pp	ppp	p	mf	mp	fff	pp	mf	mf	Pp	

Table 4: dynamics and playing modes in section A and B.

4.5. The Temporal Function

With the mapping of the dynamics and playing modes, the "outside time" architecture of *Akrata* is now complete. The last element to be discussed is the *temporal function*. Here one finds quite unambiguously the Xenakis as architect. By introducing such an eminently spatial notion as a *center of gravity* he seems to have wished to entrust this function with the role of "supporting" the elements of his composition, providing an overall "balance" in the physical sense of the word.

As has been observed previously, the abstract *outside time* structure of *Akrata* is already includes an outline of temporality in the form of two linear orderings operating at two different levels. At the macro level, it results from the "logic of succession" *between sound complexes* as induced by the group-cycles. At the micro level, it results from the

order of appearance of the elements *within* each *complex*. These two, however, do not combine into a linear ordering of the whole. Successive *sound complexes* do generally overlap so that the last event of the first will begin after the first event of the second is sounded.

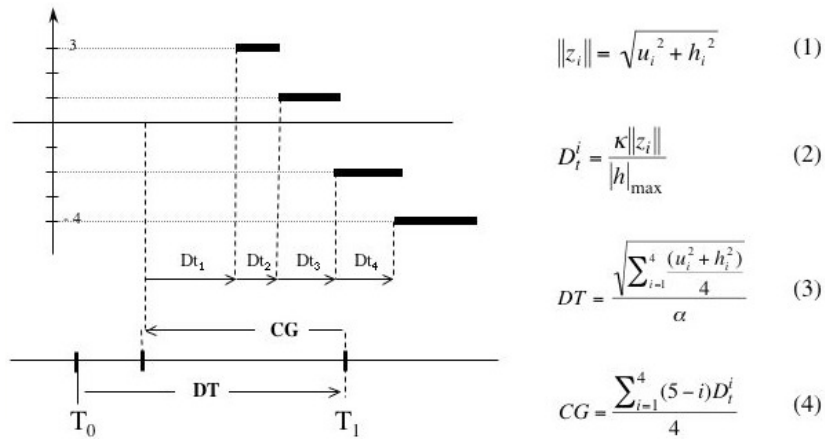


Figure 6: The temporal function (section A).

To operate the transition to the *inside time*, Xenakis defines a series of complex functions taking as “free” variable the pitch and duration values of each *sound complex*. These, determine the relative distance (*DT*) between the virtual beginning (T_i) of each *sound complex*. From these points, he subtracts the “center of gravity” (*CG*) of each *complex*. Finally, from the point thus obtained, he determines the distance to the first sound event of the complex, together with the distance separating the events that follow (D_i). The details of the computation are given and illustrated in Figure 6. The variables u and h refer to duration and pitch respectively. The function $|h|_{\max}$ appearing in (2), returns the greatest absolute value of h among the elements of the *sound complex*.

Xenakis introduces two levers into his *temporal function* in the form of two “free” variables, κ and α appearing respectively in formulas (2) and (3). Increasing the value of κ increases the distances, D_i , between events which in turn increases the value of the “center of gravity” *CG*. An increase in α , on the other hand, will shorten the distance between the “virtual beginnings” of the *sound complexes*. Thus, a decrease of the value of κ and / or an increase in that of α will increase the amount of overlapping between successive *sound complexes*. Clearly, Xenakis intended, from the very beginning, to keep control over the global outlay of his *sound complexes* in terms of distance between them and “compactness” within them. However, although κ and α are “arbitrary”, Xenakis also wanted to endow their selection with a sense of necessity. As shown in Table 5, he assigns his values according to the V_i subsets to which each element of the T-cycles belongs²¹. The values selected clearly reflect the way in which the *sound complexes* are projected on the musical surface (see section I above).

Section A																				
	I	A	A	D	G	E	B	G ²	L	D	A	L ²	E ²	G ²	B	E	D ²	L ²	A	D
	(V ₂)	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂
κ	2	2	2	1.5	1.5	2	2	2	1	1.5	2	1.5	1.5	2	2	2	2	1.5	2	2
α	2	2	2	1.5	1.5	1	2	1	2	1.5	2	1.5	1.5	1	2	1	1	1.5	2	2

²¹ With the few modifications required to accommodate sound complexes containing eight rather than just four elements, the same set of functions is applied in section B of *Akrata* as was in the first.

Section B																
	E	C	D ²	G ²	L ²	B	G	E ²	D ²	C	E	L	G	B	L ²	D
	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂	V ₃	V ₁	V ₃	V ₃	V ₂	V ₁	V ₂	V ₂
κ	2	1.5	2	2	1	1.5	1	1	2	1.5	2	2	1	1.5	1	1
α	6.5	9	6.5	6.5	5	9	5	5	6.5	9	6.5	6.5	5	9	5	5

Table 5: The parameters κ and α in sections A and B.

5. THE EVOLUTION OF THE SYSTEM AND THE UNFOLDING OF *AKRATA*

5.1. Evolution of the *sound complexes*

The “outside time” architecture, together with its *temporal function* complete the “system” determining the first section of *Akrata*. One can verify that the *sound complexes* as marked in Example 1 do indeed follow its constraints. As it has been described up to now, however, it *only* applies to section A. Over the course of the works’ unfolding, Xenakis will introduce new elements into his system, progressively moving away from the opening principles and into neighboring logical universes. Parallel to this process, one can observe a progressive simplification of the formal expressions used, together with what seems to be an increase in the “tinkering” with its determinism. This process reaches its apex in the last section of the work.

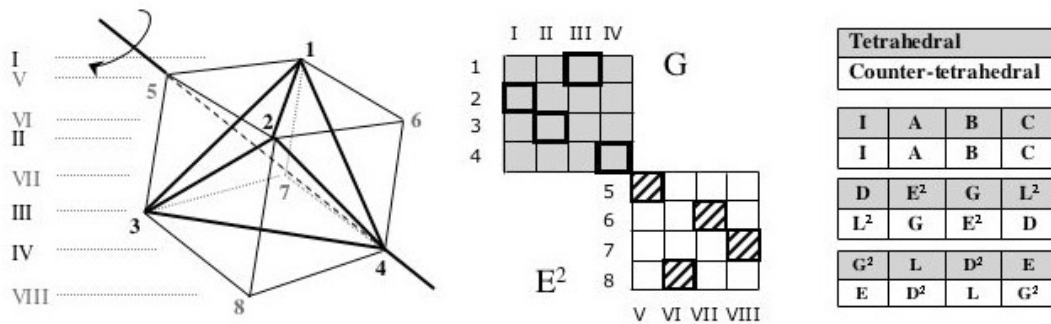


Figure 7: tetrahedron and counter tetrahedron (section B).

Section B does not yet mark a very profound alteration of the system. Rather, it increases its “complexity” by accommodating within each sound complex eight rather than just four elements. This, Xenakis obtains by inscribing his tetrahedron into a cube as shown in Figure 7. For what concerns the logic of succession, the rotations considered are still those of the original, inscribed tetrahedron. These *induce* permutations of the four remaining vertices of the cube, to which Xenakis refers as the “counter-tetrahedron”. Each element of the group T thus induces a permutation of the vertices of the tetrahedron *and* a “dual” permutation of the vertices of the counter-tetrahedron. Their correspondence is given on the right hand side of Figure 7. Strictly speaking, there are now two distinct, but dual, underlying T-cycles unfolding over the course of the section.

The projection of the rotations onto the pitch / duration plane follows closely the logic established in the previously section. Keeping both tetrahedra separate, Xenakis thus obtains two four by four grids. He combines these grids into a larger eight by eight grid by alternating their lines and columns as shown in Figure 8. This augmented grid is then mapped onto thirty-two corresponding areas delimited on the duration / pitch plane. The shaded areas are those corresponding to the tetrahedron, the white ones correspond to the counter-tetrahedron and the crosses mark the selection induced by the rotation G (Figure 8). Incidentally, by keeping the two tetrahedra separate, Xenakis also

obtains a first outline of orchestration. Indeed, the woodwinds, that appear for the first time with section B are assigned to the counter-tetrahedron, while the brass instruments continue to spell out the rotations of the main tetrahedron.

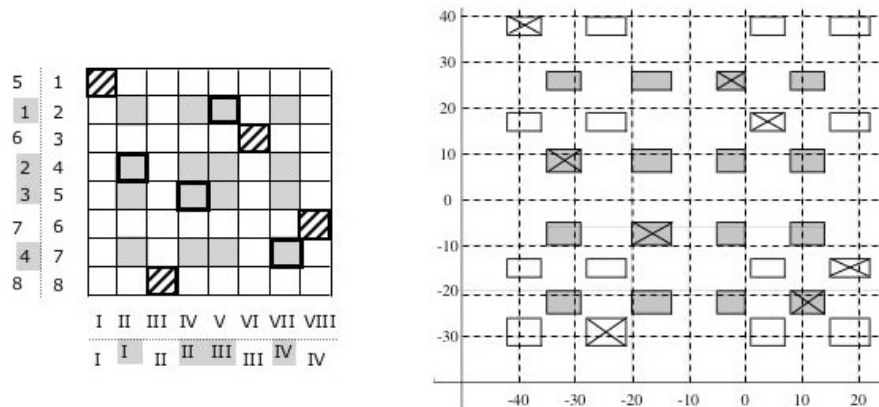


Figure 8: translation of T elements into *sound complexes* (section B).

A completely different process is established in section C. As was discussed section 3.1., it is the section of the work where the number of elements of the group T involved is the lowest: just 4 (the magical number of *Akrata*, if there has to be one), resulting in particularly short cycles. The translation into *sound complexes* of these comparatively “simple” cycles, however, goes through its first substantial alteration. According to the logic applied to section B, the counter-tetrahedron should spell out exactly the same elements as the tetrahedron (see Figure 7). Instead, Xenakis introduces the first permutations that fall outside T, thus violating for the first time the spatial integrity of his solid. While the logic of succession induced by the T-cycles continues underneath, the counter-tetrahedron is metaphorically “bent” out of three-dimensional space, so that the last two columns of its corresponding four by four grid are switched (see Figure 9). The image of “bending” the cube is actually Xenakis’, as numerous drawings and annotation in his sketches confirm. Now to each element of T, corresponds an element of S_4 , not in T, that Xenakis labels I’, A’, B’ and C’.

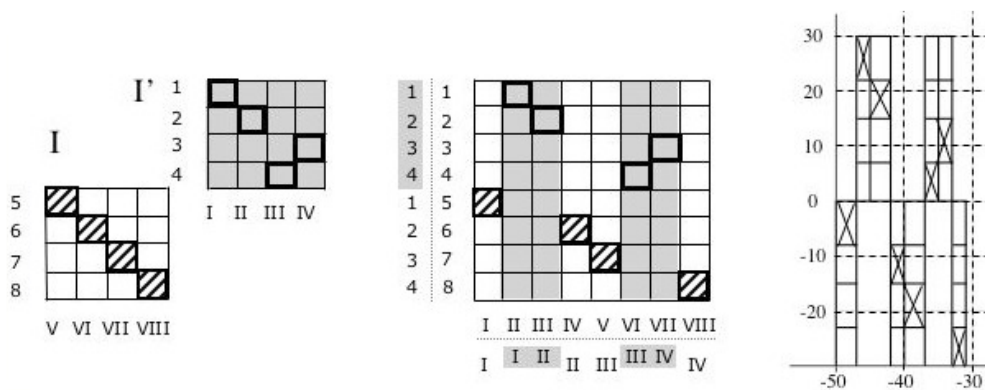


Figure 9: translation of T elements into *sound complexes* (section C).

Finally, the last section is based on two distinct underlying cycles. The first one, corresponding to the opening tutti passage spanning measures 200 to 215, is followed by a cycle in which elements appear that are not part of the group of rotations of the tetrahedron. The details of the organization of this section are, unfortunately, still unclear. The elements of S_4 that are not in T are labeled Q_1 to Q_{12} (this is Xenakis’ labeling!) without any reference to the elements

obtained previously through the “switching” of columns on the four by four grid. The new labeling most clearly prefigures the rotations used for the composition of *Nomos Alpha*. The link, however, is not as straightforward as it might seem²². Table 6 lists the permutations of the tetrahedron and counter-tetrahedron as they appear in the sketches and in the score, labeled according to the convention established in section C.

The way the permutations are projected on the pitch / duration plane, on the other hand, only differs from the previous sections in that three different “templates” are used over the course of the section. The first, reproduced on the left hand side of Figure 8, is applied exclusively to the opening tutti. The second and third are applied in alternation over the remainder of the piece. They are both reproduced on the right hand side of Figure 8: the first in solid lines, the second in dotted lines. The latter is applied whenever the underlying cycle element belongs to T (D, E, E²...), while the former is used in all other cases (Q₁₂, Q₅, Q₁₁...).

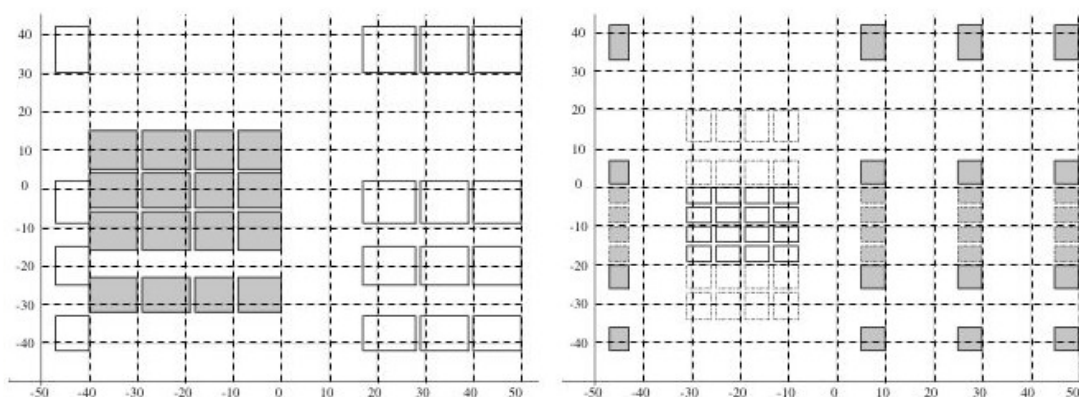


Figure 10: translation of T elements into *sound complexes* (section D).

	D	Q ₁₂	Q ₅	E	Q ₁₁	Q ₁₀	E ²	Q ₇	Q ₅	D ²	Q ₉	Q ₈	G	Q ₇	Q ₁₀	G ²	Q ₁₁	Q ₈	D	Q ₁₂
Tertrah.	L ²	D ²	G	D ²	I	L ²	G	L ²	G	E	C	D	E ²	L ²	L ²	L	I	G	L ²	D ²
Counter	D ^{2'}	A'	E'	B'	E ^{2'}	G ^{2'}	L'	C'	E'	A'	L ^{2'}	D ^{2'}	E'	C'	G ^{2'}	I'	E ^{2'}	E'	D ^{2'}	A'

Table 6: Rotations in section D.

5.2. The evolution of the sieves, intensities and playing modes

The inductive process determining the succession of sieves carries over to section B. The sieves initializing the process correspond to the last two of section A. The set of transformations, however, is slightly altered (see Table 3). Section C being built on T-cycles that engage only elements of V₁ (I, A, B and C) and section D taking the composition outside the realm of the group T (and thus of the V_i), the logic determining the sieve transformations was bound to be modified. The new criteria Xenakis adopts are also summarized in the Table 3 (in the part referring to section D, A stands for any element of T and Q for any element of S₄ not in T).

The mechanism applied to section C amounts to a drastic simplification: the only operation remaining is the transposition of the sieve up a semitone. It is applied whenever both preceding *complexes* are attached to either A, B, or

²² The fact that the permutations are defined as projections on a line makes them particularly sensitive to the choice of referential. Moreover, it is unclear whether Xenakis was thinking in terms of rotations, of permutations, or in terms of grids.

C. The sieve is carried over to the next *sound complex* without change in all other cases. A unique sieve, $\square_0 + A_1$ together with its transpositions thus permeates the entire section. The case is slightly different, but closely related in section D. There, the operations imply that either the starting sieve \square_0 , its complement, or the total chromatic spectrum appear (they do 18, 8, and 4 times respectively). Incidentally, it should be noted that the sieves are never sounded in their entirety nor by themselves, as there are overlaps between *sound complexes*.

Concerning dynamics, the same ambiguities discussed in relation to the first section also apply to the second. Table 4 shows the mapping between cycle elements and dynamics as it appears in the score. The dynamics applied to section C, a long decrescendo, with the penultimate *complex* played *fff*, seems systematic enough not to require any formal description. Concerning the playing modes, sections B and C are, respectively played normal and *staccato* all the way through. They are therefore not problematic either. Section D, on the other hand, in regard to both dynamics and “playing modes” (the latter understood in an extended sense that includes the “composite sonorities” and the various orchestration techniques) is both the most intriguing and the one whose underlying logic, provided there is one, most stubbornly resists elucidation. In section D, the parameters are no longer determined over an entire *sound complex*, but seem to have been dispersed among the eight vertices of the cube. Still, numerous elements in the sketches point towards attempts to systematize this part of the composition. It is not to be excluded that its “logic” amounts to something akin to the *kinematic diagrams*. In any case, it is this part of *Akrata* that most clearly prefigures the principles that were later applied to the composition of *Nomos Alpha*.

5.3. Applications and Mutations of the *Temporal Function*

The temporal function will turn out to be the element that will at once undergo the most drastic simplifications over the course of the work and be subject to the clearest *a posteriori* “corrections”. Table 5 above shows the values selected by the composer for section B, in relation to the underlying V_i cycle. There, the only change in the *temporal function*’s relates to accommodating eight, rather than just four, vectors per *sound complex*.

Despite the levers he built into his function, Xenakis would have felt too constrained by his function as soon as he arrived at section C. The distance between the elements of each complex is small, while the distance between *complexes* is high. His selection of values for $\square\square\square\square\square$ clearly reflects this intension (Table 7). But this was, apparently not enough, then Xenakis also modified the definition of the D_i . By adding the $|u|_{\max}$ function in the denominator, the values of the D_i ’s automatically decrease without affecting the distances, DT , between the “virtual beginnings” of the *complexes*.

$$D_i^j = \frac{2k \|z_i\|}{(|h|_{\max} + |u|_{\max})} \quad (5)$$

	I	A	A	I	I	B	B	I	I	C	C	A	B	C	A	B	C	A	B	C
κ	0.25	0.2	0.18	0.25	0.27	0.18	0.21	0.23	0.24	0.17	0.2	0.3	0.28	0.26	0.24	0.22	0.2	0.18	0.16	0.14
α	5	9	5	5	5	9	5	5	5	9	5	5	5	9	5	5	5	9	5	5

Table 7: The parameters $\kappa\square\square\square\square\alpha$ in sections C.

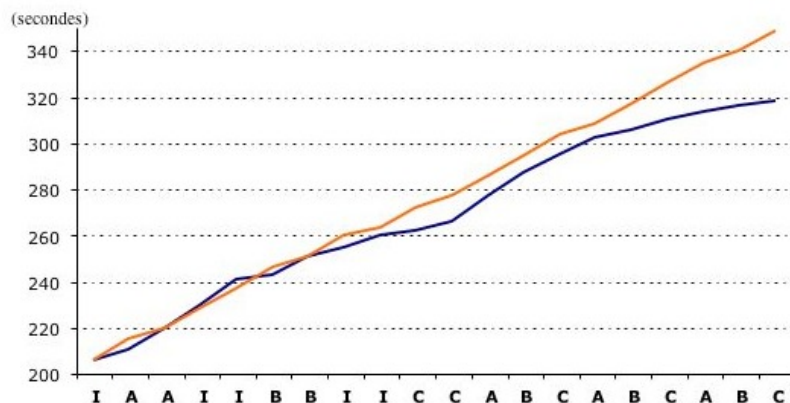
Example 2: *Akrata*, measures 301 to 315 (copyright Boosey & Hawkes Music publisher, 1968.)

In the final section D, the procedure is drastically simplified as the whole set of *functions* is reduced to just one

single formula (6). For most of the opening tutti passage, Xenakis does not even decide on any value for his new variable \square . The composition of this section was probably done without the intervention of any *temporal function*.

$$D_t^i = l \|z_i\| \quad (6)$$

The successive redefinitions and simplifications of the *temporal function* attest to what must have been either a general dissatisfaction with its results or a feeling that the work's own logic had been sufficiently assimilated to render the tedious calculations superfluous. It should be noted that Xenakis takes quite a few liberties with respect to his *temporal function* early on in his composition. The two bars silence at the very beginning of the work is already a "correction". In general, examples in which *sound complexes* have been shifted back and forth along the time axis abound. The two brief *fff* "punctuations" of measures 255 and 339, appear in complete disregard of the function's results (and indeed, also musically, as "surprises"). As last point in case, Graph 2 shows the "theoretical" beginnings (in light orange) and the "actual" beginnings (in blue) of the *sound complexes* unfolding during section C. Beyond the fact that the two lines diverge early on, it is the final rapid decrease of the distances between *sound complexes* that is of particular interest. Indeed, not only does this progression occur in contradiction to the values given by the *temporal function*, the computations relative to this particular section, an exponential progression, can actually be found elsewhere in the sketches.



Graph 2: Theoretical and the actual beginnings of the *sound complexes* in section C.

6. CONCLUSIONS

Xenakis often stressed, in his writings as well as in interviews, the need for a constant creative renewal. In his conversations with François Delalande, for instance, he stated: "each work should be as different as possible from any other one"²³. In doing so, he tended to downplay the role of any cumulative process taking place between compositions, a process that might account, at least in part, for the characteristic "sound" that permeates his oeuvre. To find such strong links between the systems underlying *Akrata* and *Nomos Alpha* is far from diminishing the import of the creative renewal within the composer's process. It does however situate the compositional means he developed, and of which we know little more than what he made public, within the continuity of a much more complex process than his writings of that period might have suggested.

²³ "Il faut que chaque pièce soit très différente, autant que possible, de toutes les autres pièces" [3] page 45. See also the preceding remarks (pages 42-45): a rare instance where Xenakis mentions a "cumulative conquest" concerning his compositional process.

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